How Do We Know What We Know?

Scientific Method

Science is first and foremost a method to determine the truth.

New theories become accepted when enough evidence is found to prove them, **not** because well respected scientists propose them.

You **become** a well respected scientist by having your theories accepted.

Examples of Paradigm Shifts

- Newtonian mechanics.
- Mendelian genetics.
- Lavoisier stoichiometry.
- Einsteinian relativity.
- Quantum mechanics.
- Plate tectonics.
- Asteroid impact caused the Cretaceous-Tertiary boundary.

Science verses Math

Science - A theory must have no known exceptions, or at least explain more of the data than any other theory.

Science verses Math

Science - A theory must have no known exceptions, or at least explain more of the data than any other theory.

Math - A proposition or Lemma should have no known exceptions.

A Theorem can be logically deduced from definitions and propositions.

Science verses Math

Science - A theory must have no known exceptions, or at least explain more of the data than any other theory.

Math - A proposition or Lemma should have no known exceptions.

A Theorem can be logically deduced from definitions and propositions.

Example: Commutative Property of Addition

$$a + b = b + a$$

Science verses Math

Science - A theory must have no known exceptions, or at least explain more of the data than any other theory.

Math - A proposition or Lemma should have no known exceptions.

A Theorem can be logically deduced from definitions and propositions.

Example: Commutative Property of Addition

$$a + b = b + a$$

Sometimes Lemmas and Theorems are interchangeable.

Assume any one and you can prove all the others.

Science verses Math

Science - A theory must have no known exceptions, or at least explain more of the data than any other theory.

Math - A proposition or Lemma should have no known exceptions.

A Theorem can be logically deduced from definitions and propositions.

Example: Commutative Property of Addition

$$a + b = b + a$$

Sometimes Lemmas and Theorems are interchangeable.

Assume any one and you can prove all the others.

Sometimes Lemmas are chosen just to see what happens.

Definitions:

An even number is an integer which is divisible by two.

An odd number is an integer which is not divisible by two.

Definitions:

An even number is an integer which is divisible by two.

An odd number is an integer which is not divisible by two.

Proposition:

The set of integers is closed under addition and multiplication.

Definitions:

An even number is an integer which is divisible by two.

An odd number is an integer which is not divisible by two.

Proposition:

The set of integers is closed under addition and multiplication.

Theorem:

Any even number can be represented as $2n$ for some integer n.

Definitions:

An even number is an integer which is divisible by two.

An odd number is an integer which is not divisible by two.

Proposition:

The set of integers is closed under addition and multiplication.

Theorem:

Any even number can be represented as $2n$ for some integer n.

Proof:

If x is an even number, then $n = x/2$ is an integer, since even numbers are divisible by 2.

Definitions:

An even number is an integer which is divisible by two.

An odd number is an integer which is not divisible by two.

Proposition:

The set of integers is closed under addition and multiplication.

Theorem:

Any even number can be represented as $2n$ for some integer n.

Proof:

If x is an even number, then $n = x/2$ is an integer, since even numbers are divisible by 2.

Multiplying both sides of the equation by 2, we get

$$2n = x$$

by the Multiplicative Property of Equality.

Definitions:

An even number is an integer which is divisible by two.

An odd number is an integer which is not divisible by two.

Proposition:

The set of integers is closed under addition and multiplication.

Theorem:

Any even number can be represented as $2n$ for some integer n.

Proof:

If x is an even number, then $n = x/2$ is an integer, since even numbers are divisible by 2.

Multiplying both sides of the equation by 2, we get

$$2n = x$$

by the Multiplicative Property of Equality.

Quod Erat Demonstrandum
("that which was to be demonstrated")

Theorem:

Any odd number can be represented as $2n + 1$ for some integer n.

Theorem:

Any odd number can be represented as $2n+1$ for some integer n.

Proof:

Part 1: $x = 2n+1$ is odd.

$$\frac{x}{2} = \frac{2n+1}{2} = n + \frac{1}{2}$$

where n is an integer. Since x is an integer and is not divisible by 2, it is odd.

Theorem:

Any odd number can be represented as $2n + 1$ for some integer n.

Proof:

Part 1: $x = 2n + 1$ is odd.

$$\frac{x}{2} = \frac{2n+1}{2} = n + \frac{1}{2}$$

where n is an integer. Since x is an integer and is not divisible by 2, it is odd.

Part 2: $x - 1 = 2n$ and $x + 1 = 2n + 2$ are even.

$$\frac{x+1}{2} = \frac{2n+2}{2} = n + 1$$

by the Distributive Property, where $n + 1$ is an integer.

Theorem:

Any odd number can be represented as $2n+1$ for some integer n.

Proof:

Part 1: $x = 2n+1$ is odd.

$$\frac{x}{2} = \frac{2n+1}{2} = n + \frac{1}{2}$$

where n is an integer. Since x is an integer and is not divisible by 2, it is odd.

Part 2: $x-1 = 2n$ and $x+1 = 2n+2$ are even.

$$\frac{x+1}{2} = \frac{2n+2}{2} = n+1$$

by the Distributive Property, where $n+1$ is an integer.

There is one and only one odd number between two consecutive even numbers, and it can be represented as $2n+1$ where n is an integer.

QED

Theorem:

The sum of two even numbers is even.

Theorem:

The sum of two even numbers is even.

Proof:

Let $x = 2n$ and $y = 2m$ represent two even numbers, where n and m are integers.

$$x + y = 2n + 2m = 2(n + m)$$

where $(n + m)$ is an integer.

Therefore $x + y$ is even. QED.

Theorem:

The sum of two odd numbers is even.

Theorem:

The sum of two odd numbers is even.

Proof:

Let $x = 2n + 1$ and $y = 2m + 1$ represent two odd numbers, where n and m are integers.

$$x + y = 2n + 2m + 2 = 2(n + m + 1)$$

where $(n + m + 1)$ is an integer.

Therefore $x + y$ is even. QED.

Theorem:

The sum of an even and an odd numbers is odd.

Theorem:

The sum of an even and an odd numbers is odd.

Proof:

Let $x = 2n$ represent an even number and let $y = 2m + 1$ represent an odd number, where n and m are integers.

$$x + y = 2n + 2m + 1 = 2(n + m) + 1$$

where $(n + m)$ is an integer.

Therefore $x + y$ is odd. QED.